Calculation the point at which a piece of box section steel will bend and not recover.

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All images were created using Google Sketchup a free 3D modelling programme from http://sketchup.google.com/

It's often the case in the metal workshop that the laws of physics are challenged in order to pursue a design aesthetic. Luckily, with some knowledge of physics and a little common sense we can employ these laws and prevent a lot of wasted time and effort.

In the case below, a square section tubular steel construction is required to support the weight of a person. It's quite clear that if a person stands on the extended tube (fig.a) the rest of the frame will tip up (fig.b)

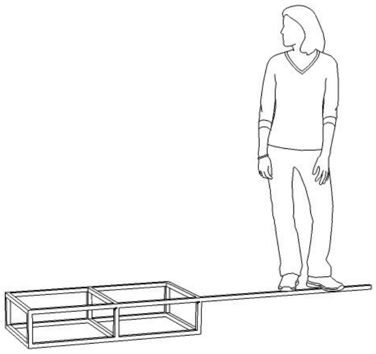

fig.a

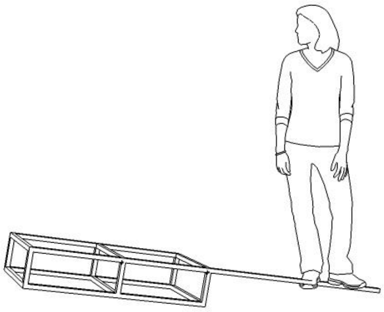

fig.b

We will need to support the tube under her weight to prevent this from happening *(fig.c)*

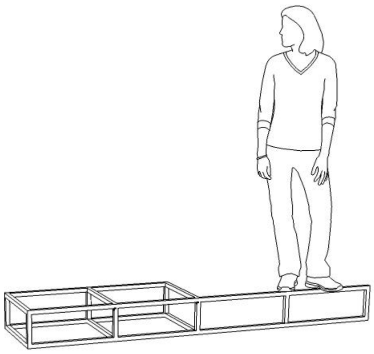

Fig.c

I know what you're thinking; that the frame could still tip up from the back and she could fall forwards. Yes; do you see how our basic understanding of the laws of physics, are deeply routed within us?

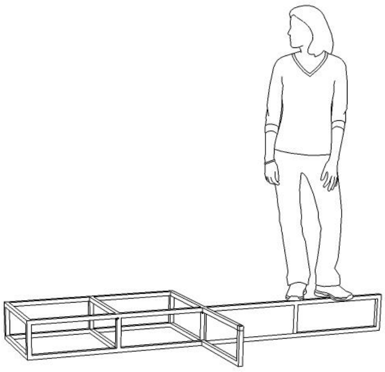

fig.d

So we have now arrived at *fig.d* a design a long way from the original, but one that will prevent the frame from tipping and be able to support the load.

However, this is not acceptable from the designer’s perspective and only *fig.a* will do.

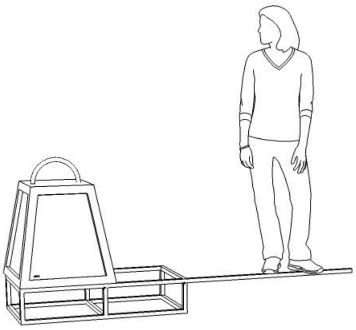

fig.e

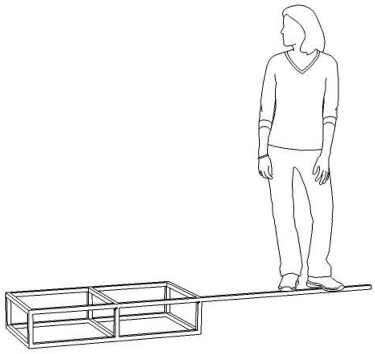

fig.f

We will need to prevent the frame from tipping and this can be done in a number of ways; by either counterbalancing the frame *(fig.e)* or bolting the frame to the floor *(fig.f*) this being the most acceptable from the designers point of view and leads us to our next problem:

What is the maximum distance the unsupported steel tube can extend from the rest of the frame before it will bend under the weight of the person standing on it?

And I suppose now we come to the essence of the document, as this simple question is not easy to answer and one that requires a much greater appreciation of the laws of physics and a determination in employing them to our advantage. Luckily the internet is a rich source of information and in particular http://www.physicsforums.com who’s members BTown and xxChrisxx did the hard work in providing me with the information necessary to answer the question.

First we must understand turning forces as it is clear that this is what we are referring to. The extended tube (acting as a lever) can no longer turn the frame and tip it up as it is bolted to the floor. However, when the woman stands on the tube the force to turn the frame is still there.

This turning force is called a ‘moment’ and is measured in Newtons per metre squared or $Nm^{2.}$ ‘Moments’ are calculated by multiplying the length of the lever (here the extended tube) by the amount or force applied to it (represented by the weight of the woman). However, weight is not the same as force. Weight is static, it has magnitude (measured in Kilograms) but it is going nowhere, whereas force (measured in newtons) has both magnitude and direction.

Fortunately, Isaac Newton can help us here: he states that 1kg falling to Earth has a force of 9.81N

Because the woman is raised up, she has a downwards force acting on the tube. Lets say she weighs 70kg (the average weight of a person) she has a force of 70 X 9.81 = 686.7N

If she stands ½ a meter away from the fixed point she exerts 0.5 x 686.7 = $343.35N/m^2$ of turning force

This helps us find the moment of turning force along any given point on the extended tube she stands on, but it doesn’t help us find out if this moment of turning force exceeds that of which the tube is capable of withstanding.

For this we require some information about the properties of the material

The Yield Strength

The yield strength or yield point of a material is defined in engineering and materials science as the stress at which a material begins to deform plastically (bend and not return). Prior to the yield point the material will deform elastically and will return to its original shape when the applied stress is removed.

"Yield (engineering)." *Wikipedia*
http://en.wikipedia.org/wiki/Yield_(engineering): 27/02/2012

There is no need to calculate this, it should be provided as a specification by the supplier of the material.

The yield strength was found here: http://www.roymech.co.uk/Useful_Tables/Matter/Strength_st.htm

The value depends on the quality of the material. In this instance the value of the yield strength was rated at 280Mpa (mega pascals) that of a construction quality steel.

Second Moment of Inertia

Also known as the **second moment area**, the **area moment of inertia**, **moment of inertia of plane area**, or **second area moment**, is a property of a cross section that can be used to predict the resistance of beams to bending and deflection, around an axis that lies in the cross-sectional plane.

"Second moment of area." *Wikipedia*
http://en.wikipedia.org/wiki/Second_moment_of_area 27/02/2012

There's no need to calculate this either as once again it should be provided as a specification by the supplier of the material (see below).

Electro Resistance Welded (ERW) Square Hollow Section Table of Dimension and Properties

Size	Thickness	Mass/m	Area of Section	Second Moment of Area	Radius of Gyration	Section Modulus	Plastic Modulus	Section Surface Area
mm x mm	mm	kg/m	cm^2	cm^4	cm	cm^3	cm^3	m^2/m
20	2	1.1	1.4	0.739	0.727	0.739	0.93	0.0748
20	2.5	1.32	1.68	0.835	0.705	0.835	1.08	0.0736
25	**2**	**1.41**	**1.8**	**1.56**	**0.932**	**1.25**	**1.53**	**0.0948**
25	2.5	1.71	2.18	1.81	0.909	1.44	1.82	0.0936
25	3	2	2.54	2	0.886	1.6	2.06	0.0923

The table above (an excerpt of http://www.roymech.co.uk/Useful_Tables/Sections/SHS_hf.html) refers to the steel tube that will be used in the final construction. The properties will change if using Rolled Hollow Section (RHS) even though the dimensions might stay the same, so it's important to make sure you have the correct table of data.

There are 3 columns that are of interest to us which are shaded in grey:

- The overall size of the tube
- The thickness of the tube wall (T)
- Second Moment of Area

You can see in bold the row which we will be referring too as we will be using 25mm x 25mm with a 2mm wall thickness.

We also need to know the distance from the centre of the tube (x-x) to the outside edge in this case 12.5mm

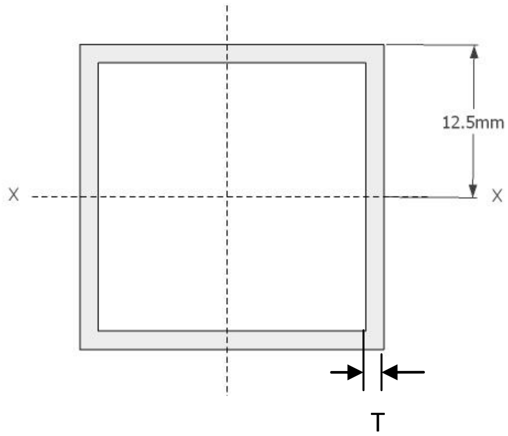

fig.g

The next step is to refer to the following equation and do some math. However, this is where my knowledge falters and I would really appreciate some input from someone who knows more about this subject than I do.

Solving the equation
Because maths symbols are not provided as standard on web browsers, in the equation below

*	has been used to represent the a multiplication symbol
x	is the unknown value we are trying to find.
/	has been used to represent the divide symbol

First let's understand the equation from a written perspective:

Yield strength of steel = (moment of turning force) * (half the material thickness /second moment of inertia)

We don't know the amount (or 'moment') of turning force required to meet that of the yield strength (bend and not return) of the steel as this is what we are trying to find out.

However, we do know that to find a 'moment' we multiply the force applied to a lever (extended tube) by the distance of the lever (here it is unknown or x). So we keep these two things apart in the equation as shown below here:

(696.7N/m^2 * x)

The remaining known values are as follows:

Yield strength of this steel	= 280Mpa
Force	= 686.7N/m^2
Distance	= x (This is what we are trying to find out)
Half the material thickness	=12.5mm
Second moment of inertia	=1.56cm^4

So the equation in numerical terms reads:

280Mpa = (686.7N/m^2 * x)*(12.5mm/1.56cm^4)

In the equation above you will notice (12.5mm ÷ 1.56cm^4) we cant divide millimetres by centimetres so we will have to convert 1.56cm^4 to mm which is 15600mm

280Mpa = (686.7N/m^2 * x)*(12.5mm÷15600mm) or in numbers alone: 280=(686.7* x)*(12.5/15600)

What we are trying to do is isolate x so we can determine what it is.

So first let’s open out some brackets by multiplying both sides of the equals sign by 15600

280*15600 = (686.7*x)12.5

Now open out the next set of brackets:

280*15600 = 686.7*x *12.5

It doesn’t matter what order we multiply things; 2*3 is the same as 3*2 as is 2*3*5 the same as 5*3*2 so let’s re-order the RHS:

280*15600=686.7*12.5*x

Now don’t forget we are trying to isolate x as this is the only way of finding out what it is, so lets make some new brackets and push x to one side:

280*15600=(686.7*12.5)*x
It hasn’t changed the balance of the equation just helped us isolate x

Now we can simplify the equation with some math before we continue: 280*15600 = 4368000 and 12.5*686.7= 8583.75 so the equation now looks like this:

4368000=(8583.75)*x

If we now divide both sides by 8583.75 we get:

4368000/8583.75=x

Some more math to solve the sum on the LHS

508.87=x

Or x (the yield point at which the metal will bend and not recover) = 508.87mm