NOTATING DISCOVERY – DISCOVERING NOTATION

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INTRODUCTION

Notation, whether mathematical or for drawing must possess an element of familiarity in order to function. I will explore later what constitutes notation, but one characteristic is linguistic – in the sense of having some kind of grammatical structure. The power of a notation language lies partly in the way it enables the user to communicate in a thought-less way, that is, by being able to avoid paying attention to some of the more commonplace aspects of the subject matter. This thought-less, or automatic character, presents a challenge: how does using such a language allow the exploration and discovery of the unknown, as opposed to the description of the already known? This question can be expressed as 'How do we notate discoveries?', but the more pressing question, in drawing if not in other domains of notation, is 'How do we discover notations?'. In the following I first clarify the nature of notation, both in mathematics and in drawing. A semiotic perspective suggests that mathematical notation is most evidently in the category of conventional signs (symbols), but that there are elements of indexical signs even in highly abstract mathematics. This indexicality is especially significant for notation in drawing, and suggests strategies for the discovery of notation as a drawing practice.

MATHEMATICAL NOTATION

NOTATION AS OTHER

The mathematical content in Bernar Venet's wall drawing *The Homology (Co-homology) Sequence of a Pair (X,A)* (2000) is inescapable. The diagram occupies a wall, extending to well above door height, there are various letters including Greek ones, superscripts, subscripts, as well as decorated letters and unusual symbols. All this, together with the technical title, leaves the viewer in no doubt about the artist's reference to mathematics. Venet's work was discussed at some length by Hofmann (2002) in a periodical for professional mathematicians where it was clear that the author considered the work to have a genuine engagement with mathematics. Some of Venet's images can be seen to be exact copies from books. In other cases there seems to be an element of collage with diagrams slightly modified by the addition of mathematical expressions found nearby in the text as a comparison of the work mentioned above with the diagram on page 247 of (Eilenberg & Steenrod, 1952) shows.

The relationship of mathematics to Venet's work is straightforward. The artist does not claim to understand the mathematical content of the diagrams, and most of them have little independent meaning once extracted from their context. Although visually striking they are similar to, say, fragments extracted from railway timetables where we have the times of trains, but the names of the stations and the dates of validity are omitted. Alternatively, they resemble ethnographic specimens in the traditional museum – here are bizarre but beautifully crafted objects assumed to have had some ritual purpose. As such, the formulæ and diagrams can be admired as examples of graphic design but are more likely to be read as evidence of another world of concepts that will be quite foreign to most viewers.

Venet presents mathematics as a spectacle of the Other. The mathematics was necessary to create the spectacle, but to claim it remains in Venet's product, would be to confuse an event with a text describing the event, or to confuse poetry with typography. The practice of mathematics is not a practice of producing written formulæ per se, it is a practice of working with certain concepts and it is understanding the relationships between these concepts that is important. The concepts, include different kinds of 'structure', in the sense explained by Barbut (1970), and in many cases the relationships are again themselves structures. The formulæ are simply a by-product – a convenient means of communicating the concepts and

their relationships.
That the study of notation itself is considered to be of little significance by most professional mathematicians is borne out by a quick (unscientific) check on the prevalence of the word 'notation' in the mathematical literature. Using the American Mathematical Society's comprehensive database of essentially all mathematical books and papers dating back over 70 years, I found only 291 items with 'notation' in the title. There were more (some 346) containing the word 'aircraft'. To give some idea of the scale of the literature involved there were over a quarter of a million items having the word 'equation' in the title. Lockhart (2002) demonstrates the danger of mistaking the ability to handle notation with understanding mathematics and presents a picture of what this would be analogous to in music or art education. In fact, mathematics is no more about notation than drawing is about graphite

MATHEMATICAL NOTATION IN PRACTICE

Notation is not absolutely fundamental in mathematics, but the practice of mathematics without notation quickly becomes severely restricted. Anyone who has tried some basic arithmetic using roman numerals instead of the familiar Arabic numerals and decimal notation will agree that notation affects calculation.
But what actually *is* notation? How can notation be distinguished from related terms such as 'sign', 'symbol', 'diagram'? Notation is something more than the use of symbols – it is their use in an organized way. Notational forms typically have a linguistic character – that is they conform to rules of syntax, and the meaning depends not just on the parts, but how they are combined together. The linguistic nature is acknowledged by Knuth (1992) p403: "Mathematical notation evolves like all languages do".
Knuth goes on to discuss two particular notations and writes (p405) of a particular formalism "… I like to have mechanical operations like this available so I can do manipulations reliably, without thinking." This power to manipulate without thinking is central to the use of mathematical notation. By encapsulating certain properties of the structures being studied within the notation itself, the mathematician is able to concentrate on other features. The discovery of new properties is facilitated by the use of notation to handle what is already known.
Unlike written verbal language, mathematical notation is rarely entirely one-dimensional. Subscripts and superscripts are of course commonplace, but beyond this, commutative diagrams from category theory, as used in Venet's work show a two-dimensional arrangement of elements in which arrows appear mediating between other symbols. This way of writing is not restricted to two dimensions, and there are often two-dimensional renditions of diagrams that belong naturally in three or more dimensions. This higher-dimensional algebra is one area in which notation plays an important role, sometimes as an aid to calculation, sometimes as a way of structuring thinking – suggesting new ideas by following the notational form that might be expected.

DRAWING NOTATION

Notation appears in drawing in a variety of ways. The one which most closely resembles the function of mathematical notation is when individual marks playing an essentially similar role to algebraic symbols in formulae are used. This is especially evident in drawings of trees and foliage and can be seen clearly, for example, in the work of Francis Towne in the 18th century (Wilcox 1997). Of this type of mark Petherbridge (2010) p113 writes that

> "Even before eighteenth-century attempts to regularize landscape into codified systems for representing leaf and tree shapes … complex sets of linear signs for depicting foliage, sky, rocks, and water were a condition of landscape sketches …".

A more conventional vocabulary of tree-signs and foliage-signs is found in architectural drawings, such as the work of Gordon Cullen in the book *Townscape* (1961).
Arscott (2006) writes of the relation of Whistler's etchings to shorthand noting that Whistler

> "introduces stuttering, stabbing mark-making which conceptually, visually and physically resembles shorthand. The marks carry concentrated meaning and rely on omission, they offer a challenge to interpretation like the baffling, meaningful squiggles of a page of shorthand, and they tell of the speed of the hand just as shorthand marks do."

This is quite different from the almost pictographic notations for foliage. These marks, unlike shorthand, may not denote a separate object in a scene but taken together provide a representation. This use of marks en masse is found also in hatching and the building up of textures and tonal areas in drawings. Petherbridge (2010) p101 refers to Henry Moore's "expressionist version of this linear code for describing volume and movement".

Connections between writing and drawing have often been examined, for example in connection with the work of Henri Michaux (de Zegher 2000, Rigaud-Drayton 2005). The notation found in the work of Towne and of Cullen mentioned above, resembles writing only marginally in a visual fashion, but presents a close similarity in the way repeatable elements are combined in a structured way to deliver a signifying whole. Notation has a relationship to writing which is not visual but operational in the sense that they work in a similar way.

Deleuze (2003, p71), adapting Bacon's term 'graph' introduces what he calls the 'diagram' as

> "the operative set of asignifying and nonrepresentative lines and zones, line-strokes and color-patches"

Can we equate notation with this kind of diagram? Deleuze (2003, p72) explains

> "Van Gogh's diagram, for example, is the set of straight and curved hatch marks that raise and lower the ground, twist the trees, make the sky palpitate, and assume a particular intensity from 1888 onwards."

The diagram is not, however, the notation itself, it seems more like the organization of the notation or the way it is structured. In the case of Van Gogh, it is surely not simply the marks themselves, but they way they relate to each other that produces the effect that Deleuze observes. So it seems appropriate to distinguish between the explicit marks which constitute notations and the higher-order structure of the implicit diagram which organizes the marks.

NOTATION AS SIGN

The role of notation in drawing is clarified by considering the semiotic nature of notation. The theory of signs developed by Peirce (Merrell, 2001) proposes three categories of sign: the icon, the symbol and the index. Any sign will refer to something but it is the manner of establishing this relationship that differs among the three classes. For an icon, the reference is produced, as Merrell (2001, p31) puts it, by "some resemblance or similarity". This is different from the symbol, where the relationship is a matter of convention. The third category of sign, the index, is particularly significant for drawing (Ashwin1984). Indices are signs which arise from the thing signified, or from its presence, examples being footprints, and also marks in drawings which can simultaneously be indices signifying the gestures which produced them and also icons or symbols signifying something quite different.

Mathematical notation generally consists of symbols which are conventional. In some elementary geometric cases there might be an iconic character, but in general the entities do not have any preferred visual form which a sign could resemble. The conventional features of some drawing is noted by Gombrich (1968): "The fact that all graphic techniques operate with conventional notation is, of course, familiar ground, …". However, this cannot be taken as a semiotic classification as many drawings exhibit marks which embody all three types of sign.

The indexical character of some mathematical notation is suggested by Mazzola and Andreatta, (2007 p24): "By means of diagrams, mathematics turns gestures into formulae". Some care is needed in interpreting this, since the diagrams they refer to mean a particular class of mathematical structures rather than arbitrary diagrams or mathematical notation in general. The diagrams in question are the 'commutative diagrams' used in category theory and appearing in Venet's work discussed earlier. These diagrams rely heavily on arrows and the naive reading of an arrow as signifying motion suggests the right kind of gesture to think of here. Reading such a diagram we can imagine hand or finger movements through space in paths that would generate the diagram. In the reverse direction gestures can be notations as in sign language or in the precise directions of the conductor of an orchestra.

Mazzola and Andreatta's view of the function of the diagram in category theory as a stage in the transformation of gesture into formulae reveals a common use of spatial analogy in mathematical structure. Even when concepts are not inherently spatial, mathematical terminology often uses space or motion through space as a metaphor. To give just two examples, one speaks of a function *from* one set *to* another, or of a module *over* a ring. This latter suggests the imposition of additional structure on the foundations of some given algebraic machinery.

DISCOVERING NOTATION

PERFORMANCE NOTATION

Notation may be used to record or to direct performances. In dance, Labanotation (Guest 2005) is one of a number of systems. Musical notations abound, although the relation of these to the activity itself can be problematic: Kuivila (2004) attributes to Ferruccio Busoni the pronouncement that “Notation is an evil separating musicians from music”. Descriptions of movements that have happened may be recorded in drawings, sequences of states may follow the conventions of the graphic novel or sequential art. Conversely, marks can be read as a score that was performed by the artist, as opposed to a representation of a sequence of events. Barthes (1979, pp112–113) recounts his own attempt, itself later repeated for a different work by Bird (2007, p499), to reproduce a work by Twombly. It is as if Barthes is enacting the death of the author by creating his own version of Twombly’s gestures. He constructs his own interpretation of the work using an indexical reading of the marks in the original.

RECORDING GESTURE

Many forms of drawing, especially those based on process might be described as systems for capturing gestures. These gestures might be deliberate movements of the body according to some scheme, or they might be movements generated as the result of some other process such as walking or travelling. They could be gestures originating in some mechanical movement such as the operation of a barograph registering atmospheric pressure or the gestures of the Earth represented as marks measuring seismic activity. Although gestures are communicating actions, we may regard machines activity as something into which we read meaning rather than something equipped with meaning by the intention of the actor.

In my own drawing practice I have been experimenting with the marks that could be obtained by using a compact digital camera while walking at night to record the traces left by moving traffic, stationary streetlights, and other light sources. By holding the camera at my side without paying attention what might be in each image and pressing the shutter at regular intervals (approximately every ten seconds), I collected (in one example) a sequence of some ninety images over the course of each walk. By repeating the same walk over fifteen days a large number of images was obtained. These could be visualized as arranged in a two-dimensional array: the sequence for each day placed in one row and the successive daily sequences placed one below the other. As the regular counting between shots ensured that the images in each sequence were taken in approximately the same sequence of places on the walk, there were fifteen images of each ‘place’.

These distinctive marks were assembled into a single image by a process of superposition (of the vertically aligned images in the two-dimensional grid) and of juxtaposition (maintaining the sequential order of the ‘places’). A small fragment of a much longer drawing appears above.

These ‘places’ are not exact geographical locations but can be loosely tied to them. It is

noticeable that some sections of the walk produce very distinctive types of mark. These might come from the terrain, for example the change to limb movements by walking uphill, or from activities in the locations, such as busier traffic where the route passed along more major roads.

NOTATIONAL DISCOVERY

The drawings just discussed can be understood as consisting of notation, but a notation that is created by a process of discovery or exploration of the route followed in the walk. The route is mapped but not using pre-determined cartographic notation to describe the territory. Instead the walk generates the notation and creates its own representation. The role of the artist in this is as an intermediary who facilitates the translation of environment to visible marks. This form of drawing has been used before, for example in the drawings made on the New York subway by William Anastasi (Lee 1999). Others have followed similar processes to Anastasi, for example Rod McLaren's drawings on the London Underground (McLaren 2003). Some of these kinds of process will lead to marks which are more clearly notational than others. They can nevertheless be seen as strategies for discovering notations, and this appears to be their particular value. By providing a tool that can conjure signs from the environment, the processes equip the artist with a way of discovering through the discovery of notation.

The evolution of mathematical notation is a slow process, as paradigms change new understandings and new frameworks suggest new kinds of notation for their description. In an individual's drawing practice, evolution cannot be left to chance mutations over decades or centuries. Evolution can however be enabled through allowing the environment to propose its own notation.

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